16. State the difference between transportation problem and assignment problem.

Transportation problem	Assignment problem
(a) Supply at any source may be any positive quantity a_i	Supply at any source (machine) will be (i.e) $a_i = 1$
(b) Demand at any destination may be any positive quantity b_j	Demand at any destination (job) will be 1 (i.e) $b_j = 1$
(c) One or more source to any number of destination.	one source (machine) to only one destination (job)

17. What do you mean by unblanced assignment problem?

If the number of rows is not equal to the number of column in the cost matrix then the given assignment problem is said to be unblanced.

18. What are the basic Characteristics of queueing system?

The input or arrival pattern. The service mechanism, the queue discipline, the customers behaviour.

19. What is Queue?

The flow of customers waiting for service rendering some service is called queue.

20. Write the Little's Formula.

The little's formula is

$$L_s = \lambda W_s, \quad L_q = \lambda W_q, \quad W_q = \rho W_q$$

21. Define a Saddle point.

A Saddle point of a pay-off matrix of a game is that position in the pay-off matrix where the maximum of the row minima coincides with the minimum of the column maxima.

22. Expand the term PERT and CPM.

PERT - Programme Evaluation Review Techtique.

CPM - Critical path Method.

23. Why dummy is added in assignment problem?

It is used to preserve the order of activities. In problem the duration of the dummy activities is zero.

24. Define two person zero sum game.

The game which has only two players and where the gain of one is the loss of the other is called two person zero sum games.

5 Marks

1. Explain the Various time estimate in PERT.

Solution:

(a) optimistic time : (t_o or a)

Optimistic time is the duration of any activity when everything goes on very well during the project. This is the minimum duration.

(b) Pessimestic time : (t_p or b)

Pessimestic time is the duration of the any activity when almost every thing goes against our will and a lot of difficulties is faced while doing a project. This is the maximum duration.

(c) Most likely time: (t_m or m)

Most likely time is the duration of any activity when Sometimes things go on very well. Sometimes things go on very bad while doing the project. This is the normal duration.

2. In a public telephone booth the arrivals are on the average 15/hour. A call on the average takes 3 min. If there is just one phone find,

(i) Expected number of callers in the booth at anytime

(ii) The proportion of the time the booth is expected to be idle.

Solution:

$$\text{Arrival Rate } \lambda = \frac{15}{60} = 1/4 \text{ / minute}$$

$$\text{service time} = 3 \text{min}$$

$$\therefore \mu = \text{service rate} = \frac{1}{3}$$

$$\therefore \rho = \frac{\lambda}{\mu}$$

$$= \frac{1/4}{1/3} = 3/4$$

(i) Expected number of callers in booth length.

$$\text{of the system} = \frac{\lambda}{\mu - \lambda}$$

$$= \frac{1/4}{1/3 - 1/4}$$

$$= \frac{1/4}{4-3/12}$$

$$= \frac{1/4}{1/12}$$

$$= 12/4 = 3$$

(ii) The proportion of the time the booth is expected to be u

$$= 1 - \rho$$

$$= 1 - 3/4$$

$$= \frac{4-3}{4}$$

$$= 1/4$$

3 In a single service queueing system $\lambda = 5$, $\mu = 6$, $N = 5$

Find P_0, L_s.

Solution:

Given

$$\lambda = 5, \mu = 6, N = 5$$

$$\rho = \frac{\lambda}{\mu} = \frac{5}{6}$$

The probability that the yard is employ is.

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$$

$$P_0 = \frac{1-5/6}{1-(5/6)^{5+1}}$$

$$= \frac{1-5/6}{1-(5/6)^6}$$

$$= \frac{1-0.83}{1-(0.83)^6}$$

$$= \frac{0.17}{1-0.32694}$$

$$= \frac{0.17}{0.67}$$

$$P_0 = 0.253$$

Average queue length.

$$L_s = P_0 \sum_{n=0}^{N} n\rho^n$$

$$= 0.253 \times \sum_{n=0}^{n} n(0.83)^n$$

$= 0.253 \times [0.83 + 2(0.83)^2 + 3(0.83)^3 + 4(0.83)^4 + 5(0.83)^5]$

$= 0.253 \times [0.83 + 1.3778 + 1.7153 + 1.8983 + 1.9695]$

$= [0.253 \times 7.7909]$

$L_s = 1.971.$

4 Write the graphical algorithm for $2 \times n$ games

step 1: plot pairs of pay off of the 'n' strategies of the players A and B on two vertical axes. Connect the pairs of points by straight lines.

step 2: Locate the highest point on the line segments that forms the lower boundry of the graph.

step 3: The lines that intersect at this point identify that strategies player B should adopt in his optimum strategy.

5 Use simplex method to solve the LPP.

maximize $Z = 4x_1 + 10x_2$

Subject to $2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

and $x_1, x_2 \geq 0$.

Solution:

The standard form is

maximize $Z = 4x_1 + 10x_2 + 0S_1 + 0S_2 + 0S_3$.

subject to

$2x_1 + x_2 + S_1 = 50$

$2x_1 + 5x_2 + S_2 = 100$

$2x_1 + 3x_2 + S_3 = 90$

and $x_1, x_2, S_1, S_2 \geq 0$

S_1, S_2, S_3 are slack variables

Table 1 :-

		C_j	4	10	0	0	0	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3	$\theta = \frac{X_B}{\text{key column}}$
0	S_1	50	2	1	1	0	0	$\theta = 50$
0	S_2	100	2	5^*	0	1	0	$\theta = 20$
0	S_3	90	2	3	0	0	1	$\theta = \frac{90}{3} = 30$
		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-4	-10	0	0	0	

x_2 is entering variable and S_2 is leaving variable

Now R_2 = old $R_2 \div 5$

$= 100 \quad 2 \quad 5 \quad 0 \quad 1 \quad 0 \quad \div 5$

New R_2 $= 20 \quad 2/5 \quad 1 \quad 0 \quad 1/5 \quad 0$

New R_1 = old R_1 − New $R_2 \times 1$

old R_1 =	50	2	1	1	0	0
New $R_2 \times 1$ (−)	20	2/5	1	0	1/5	0
	30	8/5	0	1	−1/5	0

New R_3 = Old R_3 − New $R_2 \times 3$.

Old R_3 =	90	2	3	0	0	1
New $R_2 \times 3$ (−)	60	$6/5$	3	0	$3/5$	0
	30	$4/5$	0	0	$-3/5$	1

Table II :

		c_j	4	10	0	0	0
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3
0	S_1	30	$8/5$	0	1	$-1/5$	0
10	x_2	20	$2/5$	1	0	$1/5$	0
0	S_3	30	$4/5$	0	0	$-3/5$	1
		z_j	4	10	0	2	0
		$z_j - c_j$	0	0	0	2	0

Since all $z_j - c_j \geq 0$, the soln. is optimal.

The optimal soln., is

$x_1 = 0, \; x_2 = 20$

$\text{Max } z = 4x_1 + 10x_2 = 4(0) + 10(20)$

$\text{Max } z = 200.$

6. Solve the following two person game whose pay off matrix is as follows :-

$$\begin{array}{cc} & \text{player B} \\ \text{player A} & \begin{array}{cccc} & B_1 & B_2 & B_3 \\ A_1 & 1 & 3 & 1 \\ A_2 & 0 & -4 & -3 \\ A_3 & 1 & 5 & -1 \end{array} \end{array}$$

Solution :-

$$\begin{array}{cccc|c} & \text{player B} & & & \\ \text{player A} & B_1 & B_2 & B_3 & \text{Row minimum} \\ A_1 & 1^* & 3 & 1 & 1 \\ A_2 & 0 & -4 & -3 & -4 \\ A_3 & 1 & 5 & -1 & -1 \\ \text{column maximum} & \boxed{1} & \boxed{5} & 1 & \end{array}$$

minimax = 1 , maximin = 1

∴ Saddle point = 1.

Hence, the value of the game = 1.

---

7. Assignment problem :-

$$\begin{array}{cc} & \text{jobs} \\ \text{person} & \begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ A & 8 & 4 & 2 & 6 & 1 \\ B & 0 & 9 & 5 & 5 & 4 \\ C & 3 & 8 & 9 & 2 & 6 \\ D & 4 & 3 & 1 & 0 & 3 \\ E & 9 & 5 & 8 & 9 & 5 \end{array} \end{array}$$

Solution:

Given matrix $\begin{bmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{bmatrix}$

step 1

$$\begin{bmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{bmatrix}$$

step 2.

$$\begin{bmatrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix}$$

step 3:

$$\begin{bmatrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix}$$

is given by $A \to 5,\ B \to 1,\ C \to 4,\ D \to 3,\ E \to 2$

The optimum cost is

$$= 1 + 0 + 2 + 1 + 5$$

$$= 9$$

8 Solve the assignment problem :-

$$\begin{array}{c c} & \begin{array}{cccc} I & II & III & IV \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 18 & 26 & 17 & 11 \\ 13 & 28 & 14 & 26 \\ 38 & 19 & 18 & 15 \\ 19 & 26 & 24 & 10 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 7 & 15 & 6 & 0 \\ 0 & 15 & 1 & 13 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 11 & 5 & \cancel{0} \\ \boxed{0} & 11 & \cancel{0} & 13 \\ 23 & \boxed{0} & 2 & \cancel{0} \\ 9 & 12 & 13 & \boxed{0} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & \cancel{0} & \boxed{0} \\ \cancel{0} & 11 & \boxed{0} & 14 \\ 23 & \boxed{0} & 2 & 1 \\ 9 & 12 & 13 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & \boxed{0} & \cancel{0} \\ \boxed{0} & 11 & \cancel{0} & 14 \\ 23 & \boxed{0} & 2 & 1 \\ 8 & 11 & 12 & \boxed{0} \end{bmatrix}$$

The assignment cost is

$= 17 + 13 + 19 + 16$

$= 63.$

9. Solve the assignment problem:

$$\begin{bmatrix} 1 & 4 & 6 & 3 \\ 9 & 7 & 10 & 9 \\ 4 & 5 & 11 & 7 \\ 8 & 7 & 8 & 5 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 0 & 3 & 5 & 2 \\ 2 & 0 & 3 & 2 \\ 0 & 1 & 7 & 3 \\ 3 & 2 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 2 & 2 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & 4 & 3 \\ 3 & 2 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \boxed{0} & 3 & 2 & 2 \\ 2 & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & 1 & 4 & 3 \\ 3 & 2 & \cancel{0} & \boxed{0} \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \boxed{0} & 2 & 1 & 1 \\ 3 & \cancel{0} & \boxed{0} & 2 \\ \cancel{0} & \boxed{0} & 3 & 2 \\ 4 & 2 & \cancel{0} & \boxed{0} \end{bmatrix} \end{array}$$

Assignment cost is

$A \to 1$, $B \to 3$, $C \to 2$

$D \to 4$.

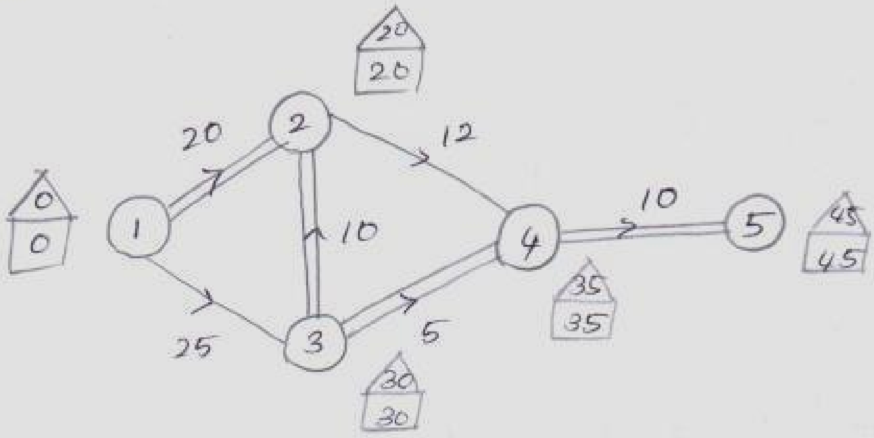
$= 1 + 10 + 5 + 5$

$= 21.$

10 Draw the network for the following activities and find the critical path.

Activity	1-2	1-3	2-3	2-4	3-4	4-5
Duration	20	25	10	12	5	10

Solution:



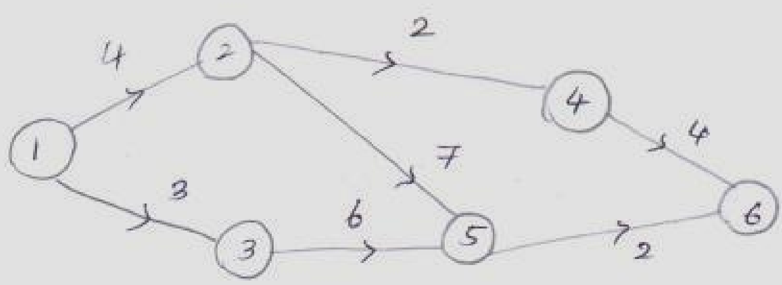
∴ the critical path is.

$$(1) \xrightarrow{20} (2) \xrightarrow{10} (3) \xrightarrow{5} (4) \xrightarrow{10} (5)$$

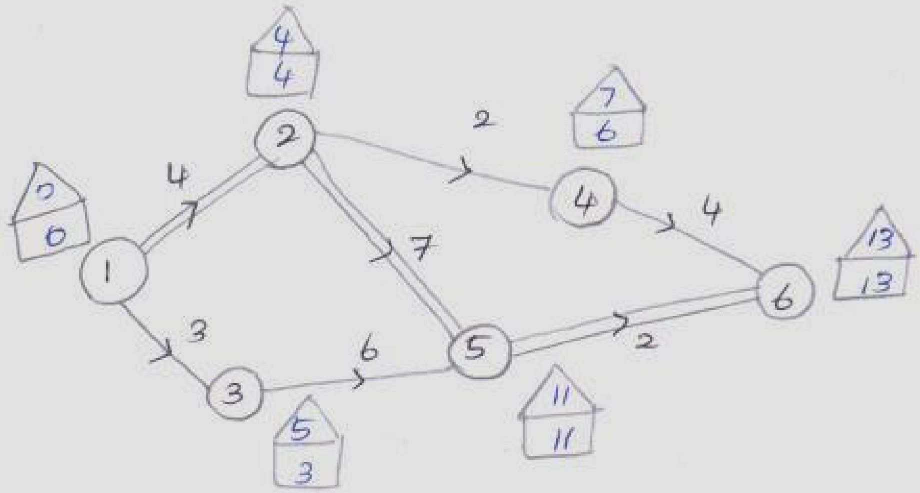
$= 20 + 10 + 5 + 10$

$= 45$.

11. Find the critical path for the following network.



Solution:



∴ the critical path is

$$① \xrightarrow{4} ② \xrightarrow{7} ⑤ \xrightarrow{2} ⑥$$

$= 13$.

12. Bring out the differences between transportation problem and assignment problem.

Transportation problem	Assignment problem
* supply at any source may be any positive quantity a_i	supply at any source will be 1 (ie) $a_i = 1$.
* Demand at any destination may be any positive quantity b_j	Demand at any destination (job) will be 1 (ie) $b_j = 1$.
* one or more source to any number of destination.	one source (machine) to only one destination (job).

13. Solve the LPP by simplex method:

Minimize : $Z = 8x_1 - 2x_2$.

Subject to : $-4x_1 + 2x_2 \le 1$

$5x_1 - 4x_2 \le 3$ and

$x_1, x_2 \ge 0$.

Sol:

The standard form is

$$\text{Max } Z^* = -8x_1 + 2x_2 + 0S_1 + 0S_2$$

subject to $-4x_1 + 2x_2 + S_1 = 1$.

$5x_1 - 4x_2 + S_2 = 3$

$x_1, x_2, S_1, S_2 \ge 0$

Table 1 :

		C_j	-8	2	0	0	
C_B	V_B	X_B	x_1	x_2	S_1	S_2	$\theta = \frac{X_B}{\text{Key column}}$
0	S_1	1	-4	2^*	1	0	$\frac{1}{2}$
0	S_2	3	5	-4	0	1	$-ve$
		Z_j	0	0	0	0	
		$Z_j - C_j$	8	-2	0	0	

New R_1 = old $R_1 \div 2$

$= 1 \quad -4 \quad 2 \quad 1 \quad 0 \quad \div 2$

New $R_1 = \frac{1}{2} \quad -2 \quad 1 \quad \frac{1}{2} \quad 0$

New R_2 = Old R_2 − New $R_1 \times -4$.

Old R_2 =	3	5	−4	0	1
New $R_1 \times -4$ (−)	2	8	−4	−2	0
New R_2	5	−3	0	2	1

Table II :

		C_j	−8	2	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
2	x_2	$\frac{1}{2}$	−2	1	$\frac{1}{2}$	0
0	s_2	5	−3	0	2	1
		Z_j	−4	2	1	0
		$Z_j - C_j$.	4	0	1	0

Here all $Z_j - C_j \geq 0$. The solution is optimum

The optimal solution is

$$x_2 = \frac{1}{2} \quad , x_1 = 0$$

$$\text{Min } Z = 8x_1 - 2x_2$$

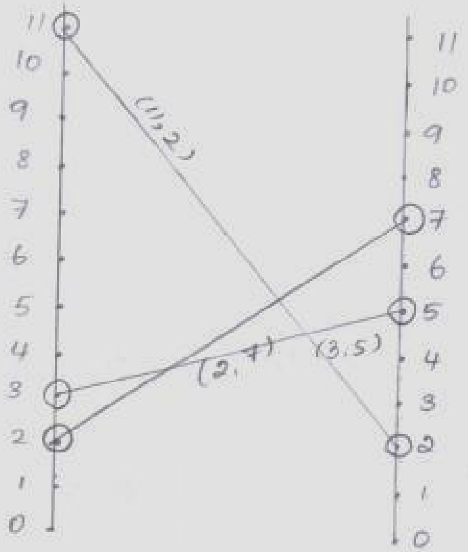
$$= 0 - 1$$

$$\text{Min } Z = -1.$$

14. Solve the following game using graphically method.

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix} \end{array}$$

solution:



The matrix reducas to 2×2

$$\begin{bmatrix} a_{11} & a_{12} \\ 2 & 7 \\ a_{21} & a_{22} \\ 11 & 2 \end{bmatrix}$$

$$\lambda = a_{11} + a_{22} - (a_{12} + a_{21})$$

$$= 2 + 2 - (7 + 11)$$

$$= -5$$

$$V = \frac{a_{11}\,a_{22} - a_{12}\,a_{21}}{\lambda}$$

$$= \frac{2 \times (2) - 7(11)}{\lambda}$$

$$= \frac{4 - 77}{-15} = \frac{73}{15}.$$

15. Obtain the initial basic feasible solution using NWC method.

			supply
3	8	5	5
5	5	3	8
7	6	9	7
4	9	5	14
Demand 7	9	18.	

$\boxed{5}$ 3	8	5	~~5~~ 0
$\boxed{2}$ 5	$\boxed{6}$ 5	3	~~8~~ 6 0
7	$\boxed{3}$ 6	$\boxed{4}$ 9	~~7~~ 4 0
4	9	$\boxed{14}$ 5	~~14~~
~~7~~	~~9~~	~~18~~	
~~2~~	~~3~~	14	
0	0		

$= 3 \times 5 + 5 \times 6 + 6 \times 3 + 9 \times 4 + 5 \times 14$

$= 15 * 30 + 18 + 36 + 70$

$= 169$

∴ The Transportation cost is 169.

10 Marks

1) Apply the principle of duality solve the LPP.

Min $Z = 2x_1 + 2x_2$

subject to

$$2x_1 + 4x_2 \geq 1$$
$$x_1 + 2x_2 \geq 1$$
$$2x_1 + x_2 \geq 1$$
and $x_1, x_2 \geq 0$

Solution:-

Given Primal LPP is

Min $Z = 2x_1 + 2x_2$

Subject to

$$2x_1 + 4x_2 \geq 1$$
$$x_2 + 2x_2 \geq 1$$
$$2x_1 + x_2 \geq 1$$
and $x_1, x_2 \geq 0$

The dual problem is

Max $W = y_1 + y_2 + y_3$

subject to

$$2y_1 + y_2 + 2y_3 \leq 2$$
$$4y_1 + 2y_2 + y_3 \leq 2$$
and $y_1, y_2, y_3 \geq 0$

Adding slack variables s_1, s_2 with the constraints we get

$$2y_1 + y_2 + 2y_3 + S_1 = 2$$
$$4y_1 + 2y_2 + y_3 + S_2 = 2$$
and $y_1, y_2, y_3, S_1, S_2, S_3 \geq 0$

The objective function is

$$Max\ W = y_1 + y_2 + y_3 + 0s_1 + 0s_2$$

1 Table :-

		b_j	0	0	0	0	-1	
C_B	y_B	x_B	x_1	x_2	S_1	S_2	R_1	θ = min
0	S_1	2	2	1	2	1	0	$2/2 = 1$
0	S_2	2	4	2	1	0	1	$2/4 = 0.5$
		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-1	-1	-1	0	0	

There are three -1's in $Z_j - C_j$

Consider first column. The least positive ratio is 0.5

The corresponds to second row.

$\therefore$ The II row is the key row. Hence key element is 4. Also y_1 is entering Variable and S_2 is leaving Variable,

In the II table the new II row is given by old row $\div$ A

New Row : $1/2$ 1 $1/2$ $1/4$ 0 $1/4$

Here all $w_j - b_j$ are ≥ 0.

IV Table:

		b_j	1	1	1	0	0
C_B	y_B	x_B	y_1	y_2	y_3	S_1	S_2
1	y_3	$2/3$	0	0	1	$2/3$	$-1/3$
1	y_2	$1/3$	2	1	0	$-1/3$	$2/3$
		z_j	2	1	1	$1/3$	$1/3$
		$z_j - c_j$	1	0	0	$1/3$	$1/3$

∴ The solution is optimal. Hence $y_2 = 2/3$, $y_3 = 2/3$, $y_1 = 0$

Also from the last table x_1 and x_2 Values are the Values under s_1 and s_2

$$\therefore S_1 = \frac{1}{3}, \quad S_2 = \frac{1}{3}$$

$$\text{Min } z = 2x_1 + 2x_2$$

$$= 2\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right)$$

$$= \frac{4}{3} \quad //$$

2) Find the maximum profit of the following assignment problem:

$$\begin{array}{c|ccccc|} & A & B & C & D & E \\ \hline 1 & 32 & 38 & 40 & 28 & 40 \\ 2 & 40 & 24 & 28 & 21 & 36 \\ 3 & 41 & 27 & 33 & 30 & 37 \\ 4 & 22 & 38 & 41 & 36 & 36 \\ 5 & 29 & 33 & 40 & 35 & 39 \end{array}$$

Solution:-

Here the largest element is 41.

∴ We have to subtract all element from 41.

$$\begin{pmatrix} 9 & 3 & 1 & 13 & 1 \\ 1 & 17 & 13 & 20 & 5 \\ 0 & 14 & 8 & 11 & 4 \\ 14 & 3 & 0 & 5 & 5 \\ 12 & 8 & 1 & 6 & 2 \end{pmatrix}$$

Now apply the same method.

No. of rows = No. of columns

Given problem is balanced.

Step 1:

subtract the smallest element in each row and form modified matrix.

$$\begin{pmatrix} 8 & 2 & 0 & 12 & 0 \\ 0 & 16 & 12 & 19 & 4 \\ 0 & 14 & 8 & 11 & 4 \\ 19 & 3 & 0 & 5 & 5 \\ 11 & 7 & 0 & 5 & 1 \end{pmatrix}$$

Step 2:

subtract the smallest element in each column and form modified matrix.

Now make the assignments as usual.

$$\begin{pmatrix} 8 & 0 & \cancel{0} & 7 & \cancel{0} \\ 0 & 14 & 12 & 14 & 4 \\ \cancel{0} & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & \cancel{0} & 5 \\ 11 & 5 & \cancel{0} & 0 & 1 \end{pmatrix}$$

Here third row is not assigned. Tick this row. In this row there is One zero. Tick this column (First column). In this column there is One assigned zero. Tick the row. Now the ticking process is Over. Draw the lines through unticked rows and ticked columns.

$$\begin{array}{c c} \begin{array}{ccccc} \checkmark & & & & \end{array} & \\ \begin{pmatrix} 8 & 0 & \cancel{0} & 7 & \cancel{0} \\ 0 & 14 & 12 & 14 & 4 \\ \cancel{0} & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & \cancel{0} & 5 \\ 11 & 5 & \cancel{0} & 0 & 1 \end{pmatrix} & \begin{array}{c} \\ \checkmark \\ \checkmark \\ \\ \\ \end{array} \end{array}$$

Look for the smallest uncovered element (ie) 4. subtrac this element from all uncovered elements and at the same time add this with intersecting elements

$$\begin{pmatrix} 9 & 0 & 0 & 8 & 0 \\ 0 & 13 & 12 & 14 & 3 \\ 0 & 11 & 8 & 6 & 3 \\ 19 & 0 & 0 & 0 & 4 \\ 11 & 4 & 0 & 0 & 0 \end{pmatrix}$$

Now make the assignments as usual.

$$
\begin{array}{cc}
\begin{array}{ccccc} \checkmark & & & & \end{array} & \\
\left(\begin{array}{ccccc}
9 & 0 & \cancel{0} & 8 & \cancel{0} \\
0 & 13 & 12 & 14 & 3 \\
\cancel{0} & 11 & 8 & 6 & 3 \\
19 & \cancel{0} & 0 & \cancel{0} & 4 \\
11 & 4 & \cancel{0} & 0 & \cancel{0}
\end{array}\right) & \begin{array}{c} \\ \checkmark \\ \checkmark \\ \\ \\ \end{array}
\end{array}
$$

Here also third row is not assigned. Tick this row. In this row there is Only one zero.

Tick this column (First column). In this column, there is Only one assigned zero. Tick the row (second row)

Ticking process is Over. Now, draw the lines through Unticked rows and ticked columns.

$$
\begin{array}{cc}
\begin{array}{ccccc} \checkmark & & & & \end{array} & \\
\left(\begin{array}{ccccc}
9 & 0 & \cancel{0} & 8 & \cancel{0} \\
0 & 13 & 12 & 14 & 3 \\
\cancel{0} & 11 & 8 & 6 & 3 \\
19 & 0 & 0 & \cancel{0} & 4 \\
11 & 4 & \cancel{0} & 0 & \cancel{0}
\end{array}\right) & \begin{array}{c} \\ \checkmark \\ \checkmark \\ \\ \\ \end{array}
\end{array}
$$

Look for the smallest uncovered element (ie.,) 3, and subtract this element from all uncovered elements. At the same time add this with intersecting elements.

Look for the smallest uncovered element (i.e) 3, and subtract this element from all uncovered elements. At the same time add this with intersecting elements.

$$\begin{pmatrix} 12 & 0 & 1 & 8 & 0 \\ 0 & 10 & 9 & 11 & 0 \\ 0 & 8 & 5 & 3 & 0 \\ 22 & 0 & 0 & 0 & 4 \\ 14 & 4 & 0 & 0 & 0 \end{pmatrix}$$

Now make the assignments.

$$\begin{pmatrix} 12 & 0 & 1 & 8 & \cancel{0} \\ 0 & 10 & 9 & 11 & \cancel{0} \\ \cancel{0} & 8 & 5 & 3 & 0 \\ 22 & \cancel{0} & 0 & \cancel{0} & 4 \\ 14 & 4 & \cancel{0} & 0 & \cancel{0} \end{pmatrix}$$

Here the assignment condition is satisfied.

$\therefore$ The optimum assignment schedule is

$$1 \longrightarrow B, \quad 2 \longrightarrow A, \quad 3 \longrightarrow E, \quad 4 \longrightarrow C, \quad 5 \longrightarrow D$$

The Maximum cost $= 38 + 40 + 37 + 41 + 35$

$= Rs. 91$

3) Solve the assignment problems

$$\begin{array}{cc|cccc}
 & & \multicolumn{4}{c}{\text{Operators}} \\
 & & I & II & III & IV \\
\hline
 & A & 10 & 5 & 13 & 15 \\
 & B & 3 & 9 & 18 & 3 \\
\text{Machine} & C & 10 & 7 & 3 & 2 \\
 & D & 5 & 11 & 9 & 7
\end{array}$$

Solution:-

$$\begin{pmatrix} 10 & 5 & 13 & 15 \\ 3 & 9 & 18 & 3 \\ 10 & 7 & 3 & 2 \\ 5 & 11 & 9 & 7 \end{pmatrix}$$

step 1:

subtracting the smallest element from each of the row we get the modified matrix.

$$\begin{pmatrix} 5 & \boxed{0} & 8 & 10 \\ \cancel{0} & 6 & 15 & \boxed{0} \\ 8 & 5 & 1 & \cancel{0} \\ \boxed{0} & 6 & 4 & 2 \end{pmatrix}$$

Step 2:-

Subtract the smallest element from each of the column we get the modified matrix

$$\begin{pmatrix} 5 & \boxed{0} & 7 & 10 \\ \cancel{0} & 6 & 14 & \boxed{0} \\ 8 & 5 & \boxed{0} & \cancel{0} \\ \boxed{0} & 6 & 4 & 2 \end{pmatrix}$$

Now we make the assignments as before

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 5 & \boxed{0} & 7 & 10 \\ 0 & 6 & 14 & \boxed{0} \\ 8 & 5 & \boxed{0} & 0 \\ \boxed{0} & 6 & 4 & 2 \end{pmatrix} \end{array}$$

$\therefore$ The assignment is as follows:

$A \rightarrow 2$, $B \rightarrow 4$, $C \rightarrow 3$, $D \rightarrow 1$

The optimal cost $= 5 + 3 + 3 + 5$

$= 16$

$= 16$ Rs

4) Solve the following transportation problem.

	1	2	3	4	supply
A	21	16	25	13	11
B	17	18	14	23	13
C	32	27	18	41	19
Demand	6	10	12	15	

Solution:

21	16	25	13 [11]	11 0	(3)	-	-	-	-
17 [6]	18 [3]	14	23 [4]	13 9 3	(3)	(3)	(4)	-	--
32	27 [7]	18 [12]	41	19 12	(9)	(9)	(9)	(9)	(8)
6 0	10 7	12	15 4 0						
(4)	(2)	(4)	(10)						
(15)	(9)	(4)	(18)						
(15)	(9)	(4)	(-)						
-	(27)	(18)	-						
-	0	(18)	-						

∴ The transportation cost:

$= 13 \times 11 + 17 \times 6 + 18 \times 3 + 23 \times 4 + 27 \times 7 + 18 \times 12$

$= 143 + 102 + 54 + 92 + 189 + 136 + 216$

$= 932$ //

5) Find P_0, P_n, L_s for the information in the queueing system $\lambda = 3$ Units per hour, $\mu = 4$ Units per hour, $N = 2$

Solution:

Given

$$\lambda = 3, \ M = 4, \ N = 2$$

$$\rho = \frac{\lambda}{\mu}$$

$$= \frac{3}{4} = 0.75$$

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-0.75}{1-(0.75)^3}$$

$$= \frac{0.25}{1-0.5625} = \frac{0.25}{0.4375}$$

$$\boxed{P_0 = 0.57}$$

$$P_n = P_0 \rho^n$$

$$P_n = (0.75)(0.75)^n$$

$$L_s = P_0 \sum_{n=0}^{N} n \rho^n$$

$$= P_0 [0 + 1 \cdot \rho^1 + 2 \cdot \rho^2]$$

$$= (0.75) \sum_{n=0}^{2} n (0.75)^n$$

$$= 0.75 [0.75 + 2 (0.75)^2]$$

$$L_s = 0.75 [0.75 + 1.125]$$

$$= 0.75 (1.875)$$

$$\boxed{L_s = 1.406}$$

b) At a railway station, Only one train is handled at a time The yard can accumulate Only two trains to wait. Arrival rate of trains is 6 per hour and the service rate is 12 per hour. Find the steady state probabilities for the Various number of trains in the system. Also find the average waiting time of the new train arriving into the yard.

Solution:

$$\lambda = 6 \text{ per hour}$$

$$\mu = 12 \text{ per hour}$$

$$N = 2.$$

$$\rho = \frac{\lambda}{\mu} = \frac{6}{12} = \frac{1}{2} \text{ per hour} = 0.5$$

In steady state

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$$

$$= \frac{1-0.5}{1-(0.5)^3}$$

$$= \frac{0.5}{1-0.125} = \frac{0.5}{0.875}$$

$$P_0 = 0.57$$

$$P_1 = P_0 \rho$$

$$= (0.57) \times (0.5)$$

$$P_1 = 0.285$$

$$P_2 = P_0 \rho^2$$

$$= (0.57)(0.5)^2$$

$P_2 = 0.142$

$P_3 = P_0 \rho^3$

$= (0.57)(0.5)^3$

$P_3 = 0.0712$

The average waiting time

$$L_s = P_0 \sum_{n=0}^{2} n\rho^n$$

$$= P_0 [0 + 1(0.5) + 2(0.5)^2]$$

$$= 0.57 [(0.5) + 2(0.5)]$$

$$= 0.57 (0.5 + 0.5)$$

$$L_s = 0.57$$

---

7. Solve the following assignment problem so as to maximize the profit.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} 32 & 38 & 40 & 28 & 40 \\ 40 & 24 & 28 & 21 & 36 \\ 41 & 27 & 33 & 30 & 37 \\ 22 & 38 & 41 & 36 & 36 \\ 29 & 33 & 40 & 35 & 39 \end{pmatrix} \end{array}$$

Solution:

Here the largest elements is 41.

∴ We have to subtract all element from 41.

$$\begin{pmatrix} 9 & 3 & 1 & 13 & 1 \\ 1 & 17 & 13 & 20 & 5 \\ 0 & 14 & 8 & 11 & 4 \\ 14 & 3 & 0 & 5 & 5 \\ 12 & 8 & 1 & 6 & 2 \end{pmatrix}$$

Now apply the same method

No. of rows = No. of columns

Given problem is balanced.

Step 1 :

subtract the smallest element in each row and form modified matrix

$$\begin{pmatrix} 8 & 2 & 0 & 12 & 0 \\ 0 & 16 & 12 & 19 & 4 \\ 0 & 14 & 8 & 11 & 4 \\ 19 & 3 & 0 & 5 & 5 \\ 11 & 7 & 0 & 5 & 1 \end{pmatrix}$$

Step 2 :

subtract the smallest element in each column and form modified matrix

$$\begin{pmatrix} 8 & 0 & 0 & 7 & 0 \\ 0 & 14 & 12 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 8 & 0 & 0 & 7 & 0 \\ 0 & 14 & 12 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 0 & 1 \end{pmatrix}$$

Now make the assignment as usual.

$$\begin{pmatrix} 8 & 0 & \cancel{0} & 7 & \cancel{0} \\ 0 & 14 & 12 & 14 & 4 \\ \cancel{0} & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & \cancel{0} & 5 \\ 11 & 5 & \cancel{0} & 0 & 1 \end{pmatrix}$$

Here third row is not assigned. Tick this row. In this row there is one zero. Tick this column (First column) In this column there is One assigned zero. Tick the row Now the ticking process is Over. Draw the lines through unticked rows and ticked columns.

$$\begin{array}{c} \checkmark \\ \begin{pmatrix} 8 & 0 & \cancel{0} & 7 & \cancel{0} \\ 0 & 14 & 12 & 14 & 4 \\ \cancel{0} & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & \cancel{0} & 5 \\ 11 & 5 & \cancel{0} & 0 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ \checkmark \\ \checkmark \\ \\ \\ \end{array}$$

Look for the smallest uncovered element (ie) 4. subtract this element from all uncovered elements and at the same time add this with intersecting elements.

$$\begin{vmatrix} 9 & 0 & 0 & 8 & 0 \\ 0 & 13 & 12 & 14 & 3 \\ 0 & 11 & 8 & 6 & 3 \\ 19 & 0 & 0 & 0 & 4 \\ 11 & 4 & 0 & 0 & 0 \end{vmatrix}$$

Now make the assignment as usual

$$\begin{array}{c c} \checkmark & \\ \left(\begin{array}{ccccc} 9 & 0 & \cancel{0} & 8 & \cancel{0} \\ 0 & 13 & 12 & 14 & 3 \\ \cancel{0} & 11 & 8 & 6 & 3 \\ 19 & 0 & 0 & \cancel{0} & 4 \\ 11 & 4 & \cancel{0} & 0 & \cancel{0} \end{array}\right) & \begin{array}{c} \\ \checkmark \\ \checkmark \\ \\ \\ \end{array} \end{array}$$

Here also third row is not assigned. Tick this row. In this row there is Only one zero.

Tick this column (First column). In this column, there is Only One assigned zero. Tick the row (second row)

Ticking process is Over. Now, draw the lines through unticked rows and ticked columns.

$$\begin{array}{c c} \checkmark & \\ \left|\begin{array}{ccccc} 9 & 0 & \cancel{0} & 8 & \cancel{0} \\ 0 & 13 & 12 & 14 & 3 \\ \cancel{0} & 11 & 8 & 6 & 3 \\ 19 & \cancel{0} & 0 & \cancel{0} & 4 \\ 11 & 4 & \cancel{0} & 0 & \cancel{0} \end{array}\right| & \begin{array}{c} \\ \checkmark \\ \checkmark \\ \\ \\ \end{array} \end{array}$$

8) Solve the transportation problem:

		1	2	3	4	supply
Shop	A	8	10	7	6	50
	B	12	9	4	7	40
	C	9	11	10	8	30
Demand		25	32	40	23	

Solution:

$8\ \boxed{25}$	$10\ \boxed{25}$	7	6	$\cancel{50}$ 25 0
12	$9\ \boxed{7}$	$4\ \boxed{33}$	7	$\cancel{40}$ 33
9	11	$10\ \boxed{7}$	$8\ \boxed{23}$	$\cancel{30}$ 23
$\cancel{25}$ 0	$\cancel{32}$ 7	$\cancel{40}$ 7	23	120

$\therefore$ The transportation cost

$= 8 \times 25 + 10 \times 25 + 9 \times 7 + 4 \times 33 + 10 \times 7 + 8 \times 23$

$= 200 + 250 + 63 + 132 + 70 + 184$

$= 899$

9) Determine an initial basis feasible solution to the following transportation problem using NWCR:

	D_1	D_2	D_3	D_4	supply
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand	6	10	15	4	35

Solution:

6 [6]	4 [8]	1	5	~~14~~ ~~8~~ 0
8	9 [2]	2 [14]	7	~~16~~ ~~14~~ 0
4	3	6 [1]	2 [4]	~~5~~ 4
~~6~~	~~10~~	~~15~~	4	35
0	~~2~~ 0	~~1~~		

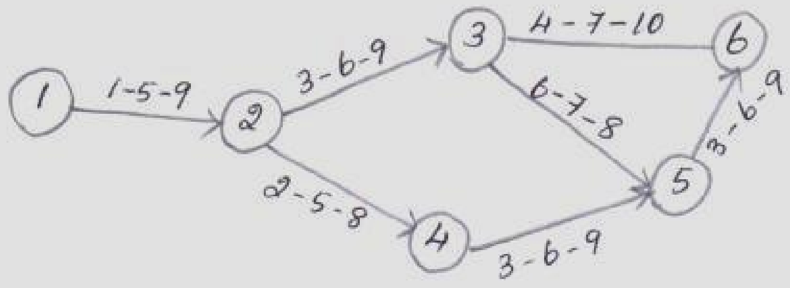
∴ The transportation cost:

$= 6\times6 + 4\times8 + 9\times2 + 2\times14 + 6\times1 + 2\times4$

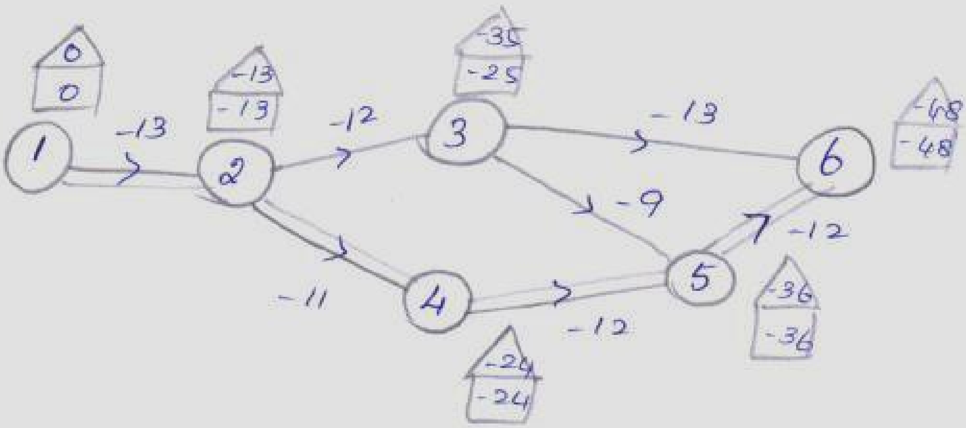
$= 36 + 32 + 18 + 28 + 6 + 8$

$= 128$

10. For the network given below find the critical path and find the probability of completing the project in 21 days.



Solution:



∴ The critical path is

= −46.